Solve It! Instruction for Mathematical Problem Solving

This research is supported by grant No. R324A070206 from the Institute of Education Sciences (IES) U.S. Department of Education.

Marjorie Montague, Ph.D.
University of Miami
Mmontague@miami.edu

www.education.miami.edu/solveit

www.education.miami.edu/solveit/lessons/video.html

Welcome!

- Introductions

- **Why the Concern about Problem Solving?**
- Consider these findings:

- Students in our schools consistently perform poorly on state (e.g., Florida Comprehensive Assessment Test), national (e.g., National Assessment of Educational Progress), and international mathematics tests (e.g., International Mathematics and Science Study).

- From 1989 to the present, the US National Council of Teachers of Mathematics (NCTM) pointed to the dismal mathematics performance of students. While the work of NCTM—particularly the NCTM standards—has helped educators improve students' mathematics learning in many ways, mathematical problem solving continues to pose significant challenges.

Goals

- To understand the *Solve It!* approach.
  - Cognitive strategy instruction
    - Teaches cognitive processes and metacognitive strategies – the content or "what" of instruction
    - Uses explicit instruction – the process or "how" of instruction
- To become familiar with the *Solve It!* approach.
  - Teaching the cognitive routine
  - Applying explicit instruction
  - Monitoring student progress

- A store sells shirts for $13.50 each. On Saturday, it sold 93 shirts. This was 26 more than it had sold on Friday. How much money did the store make on the shirts sold on Friday and Saturday?

- Rachel earns $12.50 per hour. Her company deducts 17% of her pay each week for taxes. Rachel uses the following formula: $E = 0.83(12.50h)$ to compute her earnings (E) after taxes for the hours (h) she works. What will be Rachel's earnings, after taxes, if she works 40 hours?

- You and your 3 friends are planning to go to a matinee movie on Saturday and then to a fast food restaurant for dinner. Movie tickets cost $5.00 each. A burger, fries, and soda cost $2.49 plus 5% sales tax. How much money will each person need? What will be the total cost for the group?

- Think about your students who have difficulty solving math word problems.
- What types of difficulties do these students have?
- What makes math problem solving challenging to teach?

Why *Solve It!*

- *Solve It!* has been successfully used with students—with and without disabilities—as well as those who have math difficulties in elementary, middle, and secondary schools.
- *Solve It!* has been field tested in both special education and general education classrooms.
- Teachers report that with *Solve It!* instruction, students experience immediate success and show improvement.
- Teachers find that *Solve It!* is both feasible and practical to implement in the context of the required curriculum.
- Students' self-efficacy, motivation, and interest in mathematics have been shown to improve with *Solve It!* instruction.
- Students' improved mathematics performance should have a positive impact on their middle and secondary mathematics grades, which has a significant impact on overall success in school and graduation rate, and, ultimately, on post-secondary outcomes.

Problem

A restaurant has 16 tables. Some tables seat exactly 2 people, and the rest seat exactly 5 people. The restaurant is full with 44 customers. How many tables of each size does the restaurant have?

Source: Prentice Hall Mathematics (Course 2), Page 81, Mixed Review- Question 72

- $x + y = 16$
- $x = 16 - y$
- $y = 16 - x$

- $2x + 5y = 44$
- $2x + 5\,(16-x) = 44$
- $2x + 80 - 5x = 44$
- $-3x = -36$
- $x = 12$ tables seat 2 persons and $y = 4$ tables seat 5 persons

- What's my point?

Why *Solve It!*

- Guess and check….

  Previous problem…the textbook told the students to use Guess and Check as a strategy…..Guessing is not a strategy!

- Textbook Models Do Not Work for Students with Math Problem Solving Difficulties!

  (typical 4-step model)

  Read, **decide what to do**, do it, check.

- Key words --- please do not teach key words as a strategy for solving problems… they don't work all of the time because more can mean less….next slide.

- Adam has *x* marbles. Ben gives him *y* more marbles. How many marbles does Adam have now?

- Adam has *x* marbles. Ben has *y* marbles. How many marbles do they have together?

- Adam has *x* marbles. Ben has *y* marbles more than Adam. How many marbles does Ben have?

- Adam has *x* marbles. Adam has *y* marbles less than Ben. How many marbles does Ben have?

CONCLUSIONS?

- Students, particularly students with learning difficulties, need help in understanding, analyzing, solving, and evaluating mathematical problems, primarily because no one has taught them how to think about mathematics.
- *Solve It!* helps students develop the processes and strategies used by good problem solvers by using cognitive strategy instruction.
- *Solve It!* helps teachers to understand:
  - Types of strategies used by good problem solvers.
  - Why many students are ineffective problem solvers.
  - How they can teach students to be successful problem solvers.

Solve It!

If Bob's weekly income doubled, he would be making $50.00 more than Tom. Bob's weekly income is $70.00 more than one-half of Phil's. Phil makes $180.00 a week. How much does Tom make?

Pair up

- What processes and strategies did you use to solve these problems?
- Make a list of everything you thought and did as you solved these problems.

Strategies: Definitions

- Processes that are consciously devised to achieve particular goals.
- A range of specific processes including rehearsal, outlining, memorizing, planning, visualizing.
- Cognitive and metacognitive processes or mental activities that facilitate learning and may be relatively simple or complex as a function of the level of the task and the contextual conditions.

Strategic Learning

- Students with learning difficulties (LD) may have strategy deficits or differences.
- Students may have a repertoire of strategies and yet have difficulty selecting appropriate strategies, organizing and/or executing strategies.
- They are inefficient in abandoning and replacing ineffective strategies.
- They do not readily adapt previously used strategies.
- They do not generalize strategy use.

Students with LD need

- Help in acquiring and applying cognitive processes and metacognitive strategies that underlie effective and efficient problem solving.
- To learn how to
  - understand the mathematical problems,
  - analyze the information presented,
  - develop logical plans to solve problems, and
  - evaluate their solutions.

Problem Solving/Strategic Learning

- Good PS/SL
  - Repertoire of strategies
  - Developed metacognitive abilities
  - High motivation
  - Memory capacity
  - Attentional focus
  - Developed language
  - Controlled emotions
  - Appropriately confident
  - Self-directed and self-regulating
  - Ability to generalize
- Poor PS/SL
  - Limited strategies
  - Immature metacognitive abilities
  - Low motivation
  - Working memory problems
  - Impulsive
  - Attention, memory, language problems
  - Poor self-regulation
  - Inability to detect and correct errors
  - Representation problems

Cognitive Processes and Metacognitive Strategies

Cognitive Processes

On-line mental activities that are proactive in nature (the "to do" strategies)

Successful math problem solvers…

- Read the problem for understanding.
- Paraphrase by putting the problem into their own words.
- Visualize the problem by drawing a schematic representation showing the relationship among the problem parts or making a mental image.
- Hypothesize or set up a plan for solving the problem.
- Estimate the answer.
- Compute or do the arithmetic.
- Check the process and product.

Metacognitive Strategies

Self-Regulation Strategies that require reflectivity and reactivity (the "what am I doing" and "what have I done" strategies)

Students are taught self-regulation strategies

- Say: self-instruction
- Ask: self-questioning
- Check: self-monitoring

These strategies

- help learners gain access to problem solving processes (knowledge),
- guide learners as they apply strategies (use), and
- regulate their use of strategies and their overall performance as they solve problems (control).

Cognitive Processes

RPV-HECC (Mnemonic)

- **Read** (for understanding)
- **Paraphrase** (your own words)
- **Visualize** (a picture or a diagram)
- **Hypothesize** (a plan to solve the problem)
- **Estimate** (predict the answer)
- **Compute** (do the arithmetic)
- **Check** (make sure everything is right)

Cognitive Processes and Metacognitive Strategies

Read the problem

—reading, rereading, identifying relevant/irrelevant information.

Read (for understanding)

- **Say:** Read the problem. If I don't understand, read it again.
- **Ask:** Have I read and understood the problem?
- **Check:** Check for understanding as I solve the problem.

- **Paraphrase**—translating the linguistic information by putting the problem into one's own words without changing the meaning of the story or situation.

Paraphrase (your own words)

- **Say:** Underline the important information.
  Put the problem into my own words.
- **Ask:** Have I underlined the important information? What is the question? What am I looking for?
- **Check:** Check that the information goes with the question.

- **Visualize**—transforming the linguistic and numerical information to form internal representations in memory through a drawing or image that shows the relationships among the components of a problem.

Visualize (a picture or a diagram)

- **Say:** Make a drawing or a diagram that shows the relationship among the problem parts.
- **Ask:** Does the picture fit the problem?
- **Check:** Check the picture against the problem information.

- **Hypothesize about problem solutions**—establishing a goal, looking toward the outcome, and setting up a plan to solve the problem by deciding on the operations that are needed, selecting and ordering the operations, and transforming the information into correct equations and algorithms.

Hypothesize (a plan to solve the problem)

- **Say:** Decide how many steps and operations are needed. Write the operation symbols (+, -, x, and /).
- **Ask:** If I …, what will I get? If I …., then what do I need to do next? How many steps are needed?
- **Check:** Check that the plan makes sense.

- **Estimate the outcome or answer**—validating the process as well as the product by predicting the outcome based on the question/goal and the information presented.

Estimate (predict the answer)

- **Say:** Round the numbers, do the problem in my head, and write the estimate.
- **Ask:** Did I round up and down? Did I write the estimate?
- **Check:** Check that I used the important information.

- **Compute the outcome or answer**—recalling the correct procedures for the basic operations needed for solution – calculator skills are taught/reinforced here).

Compute (do the arithmetic)

- **Say:** Do the operations in the right order.
- **Ask:** How does my answer compare with my estimate? Does my answer make sense? Are the decimals or money signs in the right places?
- **Check:** That all the operations were done in the right order.

- **Checking**—becoming aware of problem solving as a recursive activity and learning how to check both process and product by checking one's understanding and representation as well as the accuracy of the process, procedures, and computation.

Check (make sure everything is right)

- **Say:** Check the computation.
- **Ask:** Have I checked every step? Have I checked the computation? Is my answer right?
- **Check:** Check that everything is right. If not, go back. Then ask for help if I need it.

Verbal Rehearsal

- A mnemonic strategy is a memory aid. It facilitates remembering information for later recall
- A mnemonic strategy can be used to help remember steps or lists

What is the *Solve It!* mnemonic?

Mastery

RPV-HECC

Star Chart

Student Name		Mastery

Problem-solving assessment

Initial assessment and ongoing monitoring:

- measure student performance in solving mathematical problems
- ascertain each student's strategic knowledge and use of strategies
- assessment procedures that are student-centered, process-oriented, and directly relevant to the instructional program

Problem-solving assessment

- understanding a student's knowledge base, skill level, learning style, information processing, strategic activity, attitude, and motivation for learning mathematics
- the teacher is able to make judgments about both individual and group instructional needs

School: _____ Name: _____
School ID number: _____ Date: _____
Teacher: _____ Period: _____

Math Problem Solving Self-Efficacy Scale

Directions: Read the following questions carefully and then place a checkmark (√) next to the response that best describes how each statement below applies to you.

1) I am certain that I can solve math word problems.
□ Not at all true □ Hardly true □ Mostly true □ Absolutely true

2) I am confident that I can deal with solving math word problems.
□ Not at all true □ Hardly true □ Mostly true □ Absolutely true

3) Thanks to my knowledge and skills, I can handle math word problems.
□ Not at all true □ Hardly true □ Mostly true □ Absolutely true

4) I can remain calm when solving math word problems because I can rely on my knowledge and skills.
□ Not at all true □ Hardly true □ Mostly true □ Absolutely true

5) When I am confronted with a math word problem, I can find the right solution.
□ Not at all true □ Hardly true □ Mostly true □ Absolutely true

6) I can solve math word problems.
□ Not at all true □ Hardly true □ Mostly true □ Absolutely true

Math Class Grade Confidence Scale

Directions: Using the scale from **1** (*not confident at all*) to **5** (*completely confident*) in response to each question, please circle the number that best describes how confident you feel.

1 = Not confident at all
2 = A little bit confident
3 = Somewhat confident
4 = Very confident
5 = Completely confident

1	Using the scale above, *how confident are you* that you will **pass math class *at the end of this term?***	1 2 3 4 5
2	*How confident are you* that you will pass math class at the end of this term with **a grade better than a D?**	1 2 3 4 5
3	*How confident are you* that you will get **a grade of C or better?**	1 2 3 4 5
4	*How confident are you* that you will get **a grade of B or better?**	1 2 3 4 5
5	*How confident are you that you will get* **an A?**	1 2 3 4 5

DEMO

Directions: **Read** the problem. **Think** about solving the problem. Then **solve** the problem.

Date_____

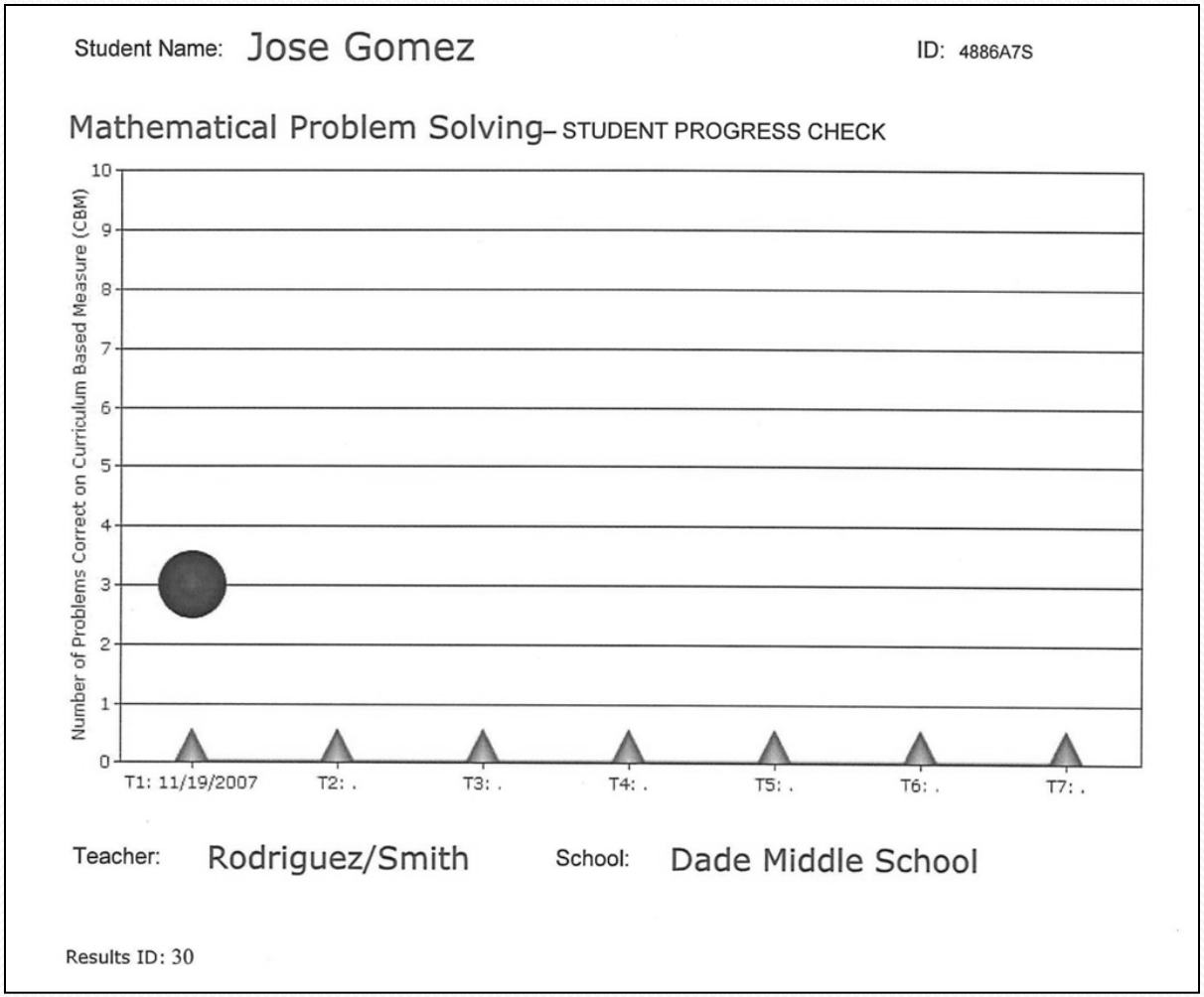
Student Name_____ Student ID # _____

Teacher's Name_____ School_____ Grade_____

1) At a waterslide, the admission charge is $5.00 for adults and $4.50 for children. Mr. Smith paid for one adult and three children. He gave the ticket seller a $20 bill. How much change did he receive?

2) The Nature Club members planned to hike a total of 200 miles. They hiked 32 miles the first day, 35 miles the second day, and 29 miles the third day. How many miles did they have left to hike?

Student Progress Graphs



Cognitive Strategy Instruction

- one of the two most powerful interventions for students with LD (Swanson, 1999)
- based in both cognitive & behavioral theory
- combines instruction in cognitive processes and metacognitive strategies
- uses verbal rehearsal and thinking aloud
- teaches students how to think and behave like successful problem solvers
- uses explicit instruction as the instructional approach

Cognitive Strategy Instruction

- Teach a problem-solving routine using guided discussion and interactive activities
- Students practice verbalizing cognitive processes and self-regulation strategies
- Students are actively engaged in the learning process
- Individual performance on a pretest determines performance goals that students understand and commit to
- Students learn to apply the processes and strategies and monitor their progress
- Students experience immediate success

Solve It!

- Which of the processes/strategies do you think will be the most difficult to teach? Why?

Activity: Modeling Verbalization of the Cognitive Routine

- Master Class Chart (RPV-HECC)
  - Group recitation
  - Individual recitation (mastery = 100% correct)
- Master Class Charts (cognitive routine)
  - Group recitation
  - Turn taking recitation

Process modeling

Process modeling is thinking aloud while demonstrating a cognitive activity.

- helps students understand how to apply the problem solving processes and strategies
- stresses learning by imitation
- provides students with the opportunity to observe and hear how to solve mathematical problems

Process modeling

- the teacher shows students how to say everything they are thinking and doing as they solve the mathematical problems
- shows students not only what to do but what not to do
- modeling of correct behaviors allows students to observe appropriate and successful application of the processes and strategies
- modeling of incorrect behaviors and responses allows students to observe what it means to locate and correct errors

Lesson 3: Solve It! Observation Checklist–Strategy Mastery Check and Practice

Teacher: _____ School: _____ Grade Level: _____
Date of Observation: _____ Time of Observation: _____
Observer: _____
Partner GA: _____ Period: _____

***SOLVE IT!* - LESSON 3**

Check the appropriate box for each instructional component:
☐ YES = The behavior is observed. ☐ NO = The behavior is not observed.

Preparation

Did the teacher:	Coding	Notes
Have student copies and transparencies of the problems?	☐ Y ☐ N	
Display Master Charts?	☐ Y ☐ N	
Distribute student folders?	☐ Y ☐ N	
Distribute cue cards?	☐ Y ☐ N	

Implementation

Did the teacher:	Coding	Notes
Check students RPV-HECC mastery?	☐ Y ☐ N	
Check that all students met 100% mastery criterion and reinforce students by checking the Star Chart?	☐ Y ☐ N	
Model problem solving using process modeling by:		
Reading the problem?	☐ Y ☐ N	
Paraphrasing?	☐ Y ☐ N	
Visualizing (emphasizing relationships among problem parts)?	☐ Y ☐ N	
Hypothesizing?	☐ Y ☐ N	

	Coding	Notes
Estimating?	☐ ☐ Y N	
Computing?	☐ ☐ Y N	
Checking?	☐ ☐ Y N	
Use the group problem solving routine?	☐ ☐ Y N	
Prompt students during student modeling?	☐ ☐ Y N	
Provide strategy rehearsal practice?	☐ ☐ Y N	
Provide positive and corrective feedback?	☐ ☐ Y N	

Notes:

Visualization (van Garderen & Montague, 2003)

- Representation process
- Drawings or diagrams that visually represent the information in the problem
- Images produced on paper or mentally
- Pictorial versus schematic representations
- Schematic or relational representations correlated with successful problem solving
- Students with LD need explicit instruction in creating schematic representations that show the relationships among the problem parts

Schematics Problem 1

Standard: MA.B.1.3.2

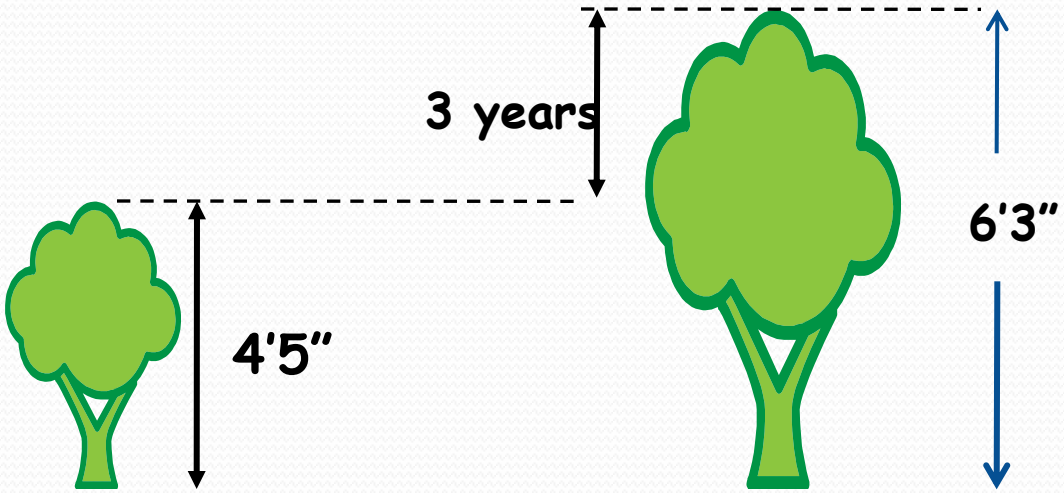
- Applies formulas for finding rates, distance, time and angle measures
- Describes and uses rates of change (for example, temperature sit changes throughout the day, or speed as the rate of change in distance over time) and other derived measures

Problem Source: Glencoe Mathematics: Mathematics Application and Concepts (Course 3) Page 173, Question #48

Problem

Three years ago, an oak tree in Emily's backyard was 4 feet 5 inches tall. Today it is 6 feet 3 inches tall. How fast did the tree grow in three years?

Schematic Representation



Answer: 7.33" in one year

Schematics Problem 2

Standard: MA.A.3.3.2

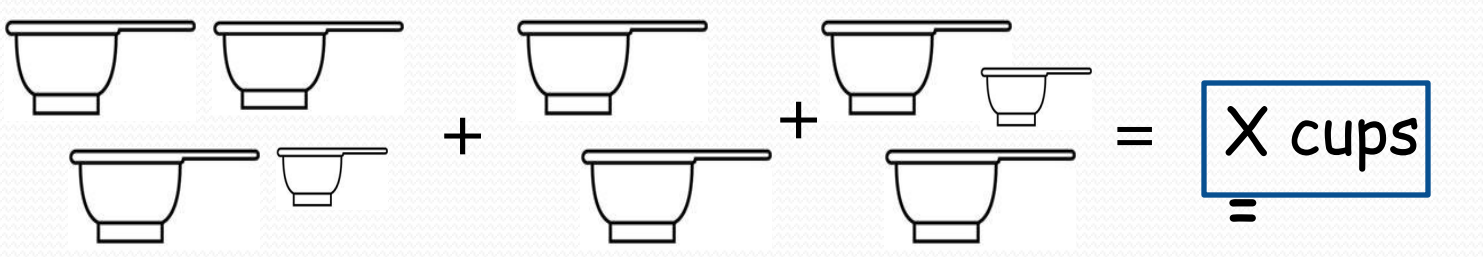
- Writes and simplifies expressions from real-world situations using the order of operations

Problem Source: Glencoe Mathematics: Mathematics Application and Concepts (Course 3) Page 203, Question #8 [FCAT]

Problem:

Suppose you made fruit punch for a party using 3 ½ cups of apple juice, 2 cups of orange juice, then 2 ½ cups of cranberry juice. How many quarts of juice did you make?

Schematic Representation



3 ½ cups apple juice + 2 cups orange juice + 2 ½ cups cranberry juice = X quarts

4 cups = 1 quart

Answer: 2 quarts

Schematics Problem 3

Standard: MA.D.2.3.2

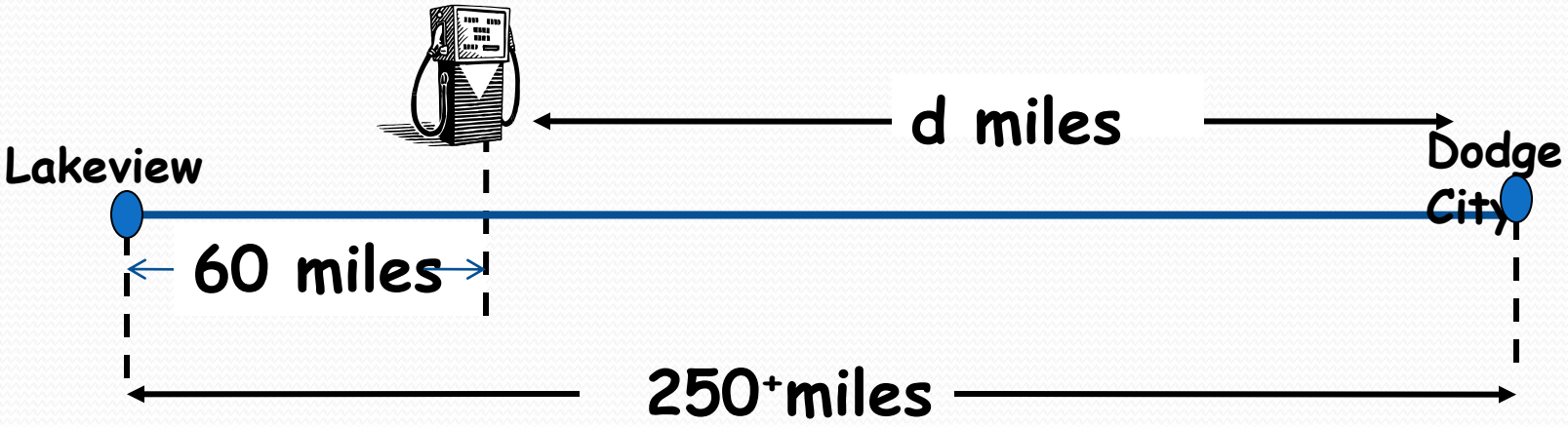
- Simplifies algebraic expressions with a maximum of two variables
- Solves single- and multi-step linear equations and inequalities that represent real-world situations.

Problem Source: Glencoe Mathematics: Mathematics Application and Concepts (Course 3) Page 499, Question #1

Problem

Michael is driving from Lakeview to Dodge City, a distance of more than 250 miles. After driving 60 miles, Michael stops for gas. Write and solve an inequality to find how much farther Michael has to drive to reach Dodge City?

Schematic Representation



Answer: d + 60 > 250; d > 190; more than 190 miles

Schematics Problem 4

Standard: MA.C.3.3.2

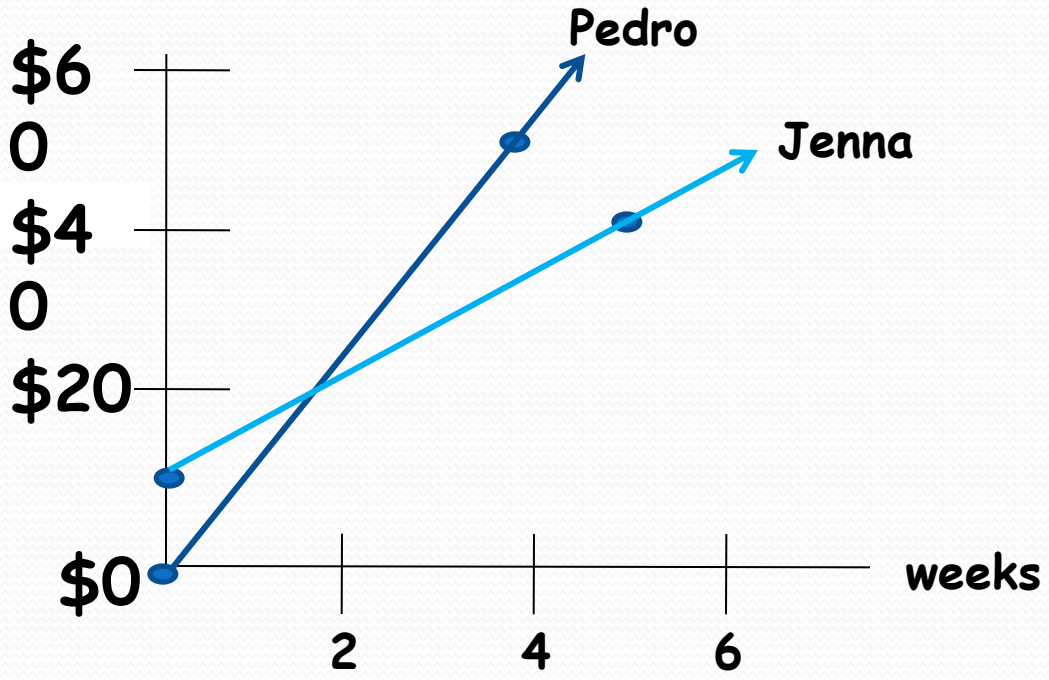
- Given an equation or its graph, finds ordered-pair solutions (for example, y = 2x).
- Given the graph of a line, identifies the slope of the line (including the slope of vertical and horizontal lines.

Problem Source: Glencoe Mathematics: Mathematics Application and Concepts (Course 3) Page 169, Question #19

Problem

Pedro and Jenna are each saving money to buy the latest video game system. Pedro's starting balance was $0.00, but by week four he had $50.00. Jenna's starting balance was $10.00 and by week five she had $40.00 saved. Who is saving more money each week?

Schematic Representation



Answer: Pedro = $12.50 per week; Jenna = $6.00 per week

Estimation (Montague & van Garderen, 2003)

- Related to number sense and conceptual understanding
- Prediction process
- Mental arithmetic
- Measurement and computational estimation
- Students generally poor at estimating
- Students with LD need explicit instruction in estimation
- More than simply rounding numbers
- Inappropriately taught in typical mathematics texts

Explicit instruction: Components

- highly structured and organized lessons but flexible (teachers can adapt the teaching routine and tailor instruction for students)
- interactive
- appropriate cues and prompts
- guided and distributed practice (role reversals)
- immediate and corrective feedback on learner performance
- positive reinforcement
- overlearning
- mastery (use graphs to monitor progress)

Explicit Instruction Procedures Definitions

Cueing/Prompting and Guided Practice

- Verbal, written, or visual reminders that guide or give direction to students to respond correctly.

Example: Ms. Williams provides appropriate cues and prompts during acquisition and application of new skills and strategies. She provides cues and prompts as students are first learning the *Solve It!* routine. Later, students will use their Student Cue Cards and Master Class Charts to guide them as they practice math problems.

Corrective Feedback

- Locating and correcting students' errors and explaining specifically what the error is and how it can be corrected.

Process Modeling

- Saying everything you are thinking and doing as you perform a task.

Example: In *Solve It!*, the teacher and then students model how successful problem solvers think and behave as they solve math problems.

Verbal Rehearsal

- Verbalizing aloud the components of a task or strategy until the components are committed to memory.

Example: In *Solve It!*, students are required to memorize the cognitive routine to 100% mastery. When they start applying *Solve It!*, they are required to think aloud as they use the cognitive routine for solving problems until they use it with facility.

Maintenance

- The ability to perform a learned task or use an acquired strategy over time.

Example: Students who learn and apply the *Solve It!* routine successfully during instruction continue to use it successfully when explicit instruction is no longer provided. Distributed practice is important to maintain performance over time.

Application:

- Ms. Williams provides practice using the *Solve It!* problems, problems from the math text, and problems from the state assessment test. She gives a test of 10 math problems every month, and students graph their score to see ongoing performance. If students begin to decline in performance, she provides a booster session to those students (pp. 63-68 of the manual).

Overlearning

- Learning a skill or behavior successfully and then completing the task or using the strategy automatically.

Example: Students who learn and apply the *Solve It!* routine successfully during instruction continue to adapt and adjust their application depending on the difficulty level of the problem without thinking about the "strategy steps."

Positive Feedback

- Explicit and positive responses to specific behaviors.

Example: Students are praised for drawing a visual representation of a math problem that shows the relationships among the problem parts. The behaviors that are praised are labeled and described.

Mastery Learning

- When the student performs a task correctly or uses a strategy appropriately and meets a certain criterion for acceptable performance.

Vignette:

- Ms. Williams monitors students' performance on the *Solve It!* progress checks. Students who achieve mastery, that is, score at least seven problems correct on at least three of four progress checks of 10 math problems, work in small groups solving novel problems while Ms. Williams continues to work with the remaining students as they work toward mastery. She knows that she may have to adjust the criterion for some students, e.g., seven of ten correct on two of the four progress checks.

Mnemonics

- Strategies or techniques that help learners remember how to perform a task or apply a strategy.

The *Solve It!* acronym, RPV-HECC, is a mnemonic strategy to help students remember the *Solve It!* routine.

Generalization

- Transfer of learning to a new task, setting, or situation.

Example: Using the *Solve It!* routine successfully in the learning disabilities resource room and then using the routine to solve problems successfully in the general education math classroom. (Students may need to be cued to use *Solve It!* in new settings, with different types of problems, and in different situations such as figuring how much money they will make per week in their after school job.)

Performance feedback

- Students are always given specific feedback regarding their performance and responses as they learn and apply the problem-solving processes and strategies.
- Performance during practice sessions and periodic progress checks is also carefully analyzed.

Performance feedback

- Students learn to appraise, critique, and monitor their own performance.
- Reinforcement by peers and the teacher for solving problems correctly and improving on the periodic progress checks.
- Use of labeled praise and directing the feedback toward the appropriate student.

Reinforcement

- essential for students who are learning problem solving
- need to know exactly which behaviors and responses are being praised so that they can be repeated
- provided with opportunities to practice giving and receiving positive feedback and praise
- shows them that they are successful and can become better problem solvers

Reinforcement

- praise must reflect an honest appraisal of students' responses
- serves to inform students that they are performing well and are making progress
- peer reinforcement for participating in practice sessions is an important part of the program
- ultimate goal is to have students recognize that they have done well and praise themselves for doing well

- Which techniques are you familiar with?
- How do you use this technique in your classroom?
- Purpose: to learn and practice instructional strategies/techniques for explicit instruction

Explicit Instruction

- Introduce the lesson (how do we make this meaningful?)
- Present content and demonstrate (how?)
- Provide guided practice with feedback (variations?)
- Provide closure (what does this mean? how?)

Thank you!